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Book Review: *Renormalization Group Theory of Macromolecules*

Renormalization Group Theory of Macromolecules. Karl F. Freed, Wiley, New York, 1987.

Polymer materials are used in an ever-increasing number of new applications and they are replacing metallic parts in many industrial and commercial components. This enormous industrial interest in polymers has given a strong impetus to applied research in this area for over four decades. However, our understanding of the basic physics of polymers was much more limited in comparison with our understanding of crystalline materials. This situation was improved about a decade or so ago because of two major breakthroughs in polymer physics. The first important advance was the realization that the macroscopic properties of long-chain molecules in solutions have the same type of scale-invariance properties as systems near a critical point. This opened the way for applications of some of the modern techniques of statistical physics and critical phenomena to polymers, including series expansions and Monte Carlo simulations. The next crucial advance was the remarkable discovery by P. G. de Gennes that polymers can be described by a field theory derived by formally taking the zero-component limit of the n-component field theory. This observation provided a formal connection to critical phenomena and the renormalization group (RG) and prompted vigorous theoretical and experimental investigations of polymers. By the late 1970s extensive RG descriptions of linear polymers, branched polymers, and the vulcanization process were developed using the field theory analogy.

It was first pointed out by de Gennes that because the field theory analogy is only formal, it would be desirable to develop RG formulations that could be applied directly to polymer conformations. Thus, one of the avenues of research in the early 1980s was the development of RG techniques that were *not* based on the field theory analogy. An example of this type of treatment is the topic of the present book, the so-called conformation-space RG developed at Chicago by Freed and his co-workers. This

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book is an exposition of the basic concepts of conformation-space RG. It is a welcome addition to the too small collection of books on applications of modern methods of statistical physics to polymers.

The main purpose of this book is to provide an introduction to conformation-space RG and its applications to polymers. The reader is first introduced to polymers with minimal digression on unessential details. The first chapter provides some motivation for the use of the universality concept in polymers and introduces the concept of coarse graining that is essential in understanding any type of RG. The most popular models of polymers, including the random-flight model, the Gaussian chain, the steric hindrance model, and the stiff chain model, are described in Chapter 2.

Chapter 3 is devoted to a discussion of the path integral formulation of noninteracting linear polymer models. The path integral formulation of the polymer chain, first introduced by Sam Edwards, is one of the most useful and intuitively satisfying methods for studying linear polymers. No matter what sort of random walk model one starts with, one must eventually take the continuum limit. One of the advantages of the path integral method is that it *is* a continuum formulation and there is no need to take the continuum limit later. The example of a flexible chain in the presence of a field is used to show how the continuous chain ideas lead to distribution functions that satisfy the diffusion equation and establish the connection with the path of a Brownian particle.

A suitable model of the configurational statistics of a polymer chain must include the interaction between different chain segments. This topic is discussed in Chapter 4. The concept of universality, however, implies that the long-wavelength properties of an isolated polymer in a good solvent, which is the subject under consideration in this book, do not depend on the details of the interactions. Thus, in treating the single-chain polymer, the interaction potential can be approximated by the excluded-volume interaction, which is a two-body delta-function pseudopotential. Although the excluded-volume model can be defined easily, it has so far eluded attempts to obtain an analytic solution and several methods have been developed to study it approximately. Most notable among these techniques are the Flory theory, the Edwards self-consistent-field method, series expansions, and Monte Carlo calculations, which are all treated in Chapter 4. The connection with critical phenomena and the intuitive—but powerful—concept of "blob," which was the most important tool in the early treatments of the scaling properties of the excluded-volume problem, are also briefly discussed at the end of Chapter 4.

The basic assumption in the scaling theory is that the physical properties of a long polymer chain in a solution depend only on a single length scale, i.e., the system is scale-invariant. This assumption immediately leads to scaling formulations that are now well known in the theory of critical phenomena. In Chapter 5 applications of the scaling ideas in polymers are presented in a manner that is readily accessible to those unfamiliar with these ideas from critical phenomena.

The basic concepts of the conformation-space RG are developed in Chapter 6. The chapter begins with an intuitive introduction to RG and ends with the derivation of the RG equations. An important result that is also discussed is the fact that the scaling laws derived in Chapter 5 emerge naturally from the RG formulation. The mathematical method of calculating exponents is discussed in Chapter 7, where the $\varepsilon = 4 - d$ expansion is discussed. Some specific examples are also given in this chapter. In particular, the partition function, the mean-square radius, and the second virial coefficient are evaluated to the lowest nontrivial order.

One of the most important applications of the RG formulation is in the study of crossover from one type of scaling behavior to another as a function of an appropriate physical parameter. In the polymer problem there is a crossover from the excluded volume to the Gaussian chain as the strength of the interaction is decreased. The RG formulation describes the scaling behavior at the two limits, but between these limits the system is described by a crossover function that reduces to the correct scaling form at the two limiting cases. In contrast to the scaling behavior, the crossover function cannot be obtained by the RG formulation. However, Freed discusses the idea that if one treats the parameters in the model phenomenologically and relates them to the experimental parameters, then it is possible to evaluate the crossover functions. This type of analysis and the specific calculations of the crossover function are described in Chapter 8. The extent to which this approach can be successful is discussed in Chapter 10, where the results are compared with experiments.

The conformation-space RG described in Chapters 6–8 is only valid for the description of an isolated polymer chain in a solvent. Treatment of higher polymer concentrations would require taking the intrachain and interchain interactions into account. Ohta and Oono have shown that a combination of Edward's screening theory of polymer solutions and the conformation-space RG can be used to develop an RG for concentrated polymer solutions. This approach is described in Chapter 9.

A topic of considerable interest is the behavior of a polymer chain at the so-called theta point, which corresponds to the condition where the steric repulsion is just balanced by attractive interactions and the excluded-volume parameter vanishes. However, at that point, three- and higher body interactions become relevant and must be taken into account. Using the critical phenomena analogy, de Gennes has shown that the theta point corresponds to a tricritical point. Thus, the RG formulation for the theta point must correspond to a tricritical-point RG analysis similar to tricritical theories in thermal phase transitions. Although there exists universal agreement among workers regarding the good solvent results, a similar consensus does not exist for the theta-point problem, particularly in $d=2$. The discussion of the theta point in Chapter 11 makes it clear that the theta-point problem is still an open question.

Despite the recent progress in polymer physics, very few books have been published on applications of modern techniques of statistical physics to polymers. De Gennes' classic book, *Scaling Concepts in Polymers Physics*, covers a very broad spectrum of topics and is of course the definitive reference book in the field. Considering the enormous impact of RG on recent advances in polymer physics, this first book that treats only applications of RG to polymers is certainly a welcome complement to de Gennes' book. The strength of this book, which is also its main weakness, is the depth to which it covers a particular type of RG, namely the conformation-space RG. Several other direct RG techniques have been developed and it would have improved this book if they had been mentioned and the similarities and differences among them pointed out. Overall, however, this book is well written and provides a sufficiently complete treatment of the topics that it does cover to make it useful for anyone interested in learning about applications of RG to polymers and conformation-space RG in particular.

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